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$$\therefore \frac{1+at+bu}{\sqrt{[1+a^2+b^2]}} = \frac{1+ct+du}{\sqrt{[1+c^2+d^2]}} = \frac{1+ft+gu}{\sqrt{[1+f^2+g^2]}} \text{ determines } t \text{ and } u.$$

$$e^2 = \tan^2 \theta = \frac{(t-a)^2 + (u-b)^2 + (au-bt)^2}{(1+at+bu)^2}, \quad e_1^2 = \frac{(t-a)^2 + (u+b)^2 + (au+bt)^2}{(1+at-bu)^2},$$

$$e_2^2 = \frac{(t+a)^2 + (u-b)^2 + (au+bt)^2}{(1-at+bu)^2}, \quad e_3^2 = \frac{(t+a)^2 + (u+b)^2 + (au-bt)^2}{(1-at-bu)^2}.$$

145. Proposed by FRANK GIFFIN, Graduate Student, State University, Boulder, Col.

If A and B be the points of contact, upon two circles X and Y , of tangents drawn from any point of their circle of similitude, then the tangent from A to Y is equal to the tangent from B to X . [From *Casey's Sequel to Euclid*, Part I., page 144.]

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let P be any point on the circle of similitude, AP, BP the tangents from P to X and Y , respectively. AD, BC the tangents from A and B to Y and X , respectively.

Let $AX=R, BY=r$. Let $AD=a, BC=b, AP=c, BP=d, PX=m, PY=n$.

$\angle APX = \angle BPY$ since P is on circle of similitude. $\therefore \angle APY = \angle BPX = \theta$.

Also, $c:d=R:r$. $\therefore d=cr/R \dots (1)$.

$m:n=R:r$. $\therefore n=mr/R \dots (2)$.

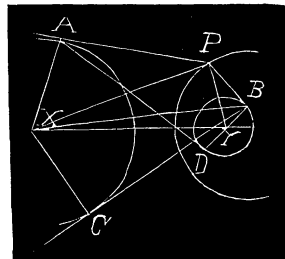
$$a^2 = AY^2 - r^2 = c^2 + n^2 - 2cn \cos \theta - r^2 \\ = c^2 + m^2 r^2 / R^2 - (2cmr/R) \cos \theta - r^2 \dots (3).$$

$$b^2 = RX^2 - R^2 = d^2 + m^2 - 2dm \cos \theta - R^2 = c^2 r^2 / R^2 + m^2 - (2cmr/R) \cos \theta - R^2 \dots (4).$$

$$(3) - (4) \text{ gives } R^2(a^2 - b^2) = (c^2 - m^2 + R^2)(R^2 - r^2).$$

$$\text{But } c^2 + R^2 = m^2.$$

$$\therefore a^2 - b^2 = 0. \quad \therefore a = b.$$



CALCULUS.

106. Proposed by M. C. STEVENS, M. A., Professor of Higher Mathematics, Purdue University, Lafayette, Ind.

$$\int_0^\pi \frac{\cos rx dx}{1-2a \cos x + a^2} = \frac{\pi a^r}{1-a^2}.$$

[Williamson's *Integral Calculus*, 6th Edition, page 174.]

Solution by WILLIAM HOOVER, A.M., Ph.D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

In Todhunter's *Plane Trigonometry*, 3d Edition, Art. 309, we have

$$\cos \alpha + a \cos(\alpha + \beta) + a^2 \cos(\alpha + 2\beta) + \dots + a^{n-1} \cos[\alpha + (n-1)\beta]$$

$$= \frac{\cos \alpha - a \cos(\alpha - \beta)}{1 - 2a \cos \beta + a^2} \dots (1),$$

if n be indefinitely increased. Make $\alpha=0$. Then

$$1 + a \cos \beta + a^2 \cos 2\beta + \dots a^{n-1} \cos[(n-1)\beta] = \frac{1 - a \cos \beta}{1 - 2a \cos \beta + a^2} \dots (2).$$

Multiply (2) by 2 and subtract a unit from both members of the resulting equation, then

$$\frac{1 - a^2}{1 - 2a \cos \beta + a^2} = 1 + 2a \cos \beta + 2a^2 \cos 2\beta + \dots 2a^{n-1} \cos[(n-1)\beta] \dots (3).$$

$$\begin{aligned} \text{Then, } \int_0^\pi \frac{\cos rx dx}{1 + 2a \cos x + a^2} &= \int_0^\pi \frac{\cos rx}{1 - a^2} \{1 + 2a \cos x + 2a^2 \cos 2x + \dots \\ &+ 2a^{n-1} \cos[(n-1)x]\} dx = \frac{2a^r}{1 - a^2} \int_0^\pi \cos^2 r x dx = \frac{\pi a^r}{1 - a^2}, \end{aligned}$$

the terms of the series but this one vanishing between the given limits, as may be seen after reducing for a few terms.

Also solved by G. B. M. ZERR, W. W. LANDIS, J. SCHEFFER, and L. C. PLANT.

107. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in Irving College Mechanicsburg, Pa.

The speed of signaling in submarine telegraph-cable varies as $x^2 \log(1/x)$, in which x is the ratio of the radius of the core to that of the covering. Prove that the *maximum speed* is attained when this ratio is $1:\sqrt{e}$.

I. Solution by J. W. YOUNG, Cornell University, Ithaca, N. Y.; COOPER D. SCHMITT, A. M., University of Tennessee, Knoxville, Tenn.; and G. B. M. ZERR, A. M., Ph. D., The Temple College, Philadelphia, Pa.

$$\text{If } y = x^2 \log\left(\frac{1}{x}\right) = -x^2 \log x, \quad \frac{dy}{dx} = -2x \log x - x = -x(\log x^2 + 1).$$

$\log x^2 + 1 = 0$ gives the maximum.

$$\therefore \log \frac{1}{x^2} = 1. \quad \therefore \frac{1}{x^2} = e, \text{ and } x = \frac{1}{\sqrt{e}}.$$

II. Solution by H. C. WHITAKER, Ph.D., Manual Training School, Philadelphia, Pa.; W. W. LANDIS, A.M., Dickinson College, Carlisle, Pa.; J. O. MAHONEY, B. E., M. Sc., Central High School, Dallas, Tex.; ALOIS F. KOVARIK, Decorah Institute, Decorah, Ia.; and J. JCHEFFER, A. M., Hagerstown, Md.

$$\text{If } S = cx^2 \log \frac{1}{x}, \quad \frac{dS}{dx} = 2cx \log \frac{1}{x} - cx.$$